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13. ABSTRACT (Maximum 200 words)

Many problems in linear prediction, signal processing, digital filtering and in several other areas can be formulated in terms of structured matrices and their inverses. Most of the algorithms which respect the structure in Matrices suffer from propagation of round-off errors. Hence all the prevalent mathematical software tools explore structure ignoring methods. We studied the problem of designing fast and numerically accurate algorithms which respect the partial structure in matrices. We also studied the problem of extending and applying the structured matrix computations to problems in $\mathcal{H}_\infty$ filtering, inverse scattering, adaptive filtering and recursive updates. We also looked into the development of robust estimation schemes for data fusion scenarios and to study the performance limits of several adaptive schemes. We have also studied how structured matrix factorizations can be used

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to develop new structures for sub-band adaptive filtering. The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ approach to controller design is an attempt to incorporate optimal performance and guarantee robustness. arguably the two most desirable properties. into a single controller. The robust performance problem formulated in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ framework largely remains an open problem. In this study. using a number of ideas from convex optimization theory, we have developed an efficient numerical approach to design fixed order mixed controller. In another study, we looked into the problem of designing equalizers for communication channels from an H_∞ point of view.

Brief Statement of the problem studied

Many problems in linear prediction, signal processing, digital filtering and in several other areas can be formulated in terms of structured matrices and their inverses. Most of the algorithms which respect the structure in Matrices suffer from propagation of round-off errors. Hence all the prevalent mathematical software tools explore structure ignoring methods. We studied the problem of designing fast and numerically accurate algorithms which respect the partial structure in matrices. We also studied the problem of extending and applying the structured matrix computations to problems in $\mathcal{H}_\infty$ filtering, inverse scattering, adaptive filtering and recursive updates. We also looked into the development of robust estimation schemes for data fusion scenarios and to study the performance limits of several adaptive schemes. We have also studied how structured matrix factorizations can be used to develop new structures for sub-band adaptive filtering. The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ approach to controller design is an attempt to incorporate optimal performance and guarantee robustness, arguably the two most desirable properties, into a single controller. The robust performance problem formulated in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ framework largely remains an open problem. In this study, using a number of ideas from convex optimization theory, we have developed an efficient numerical approach to design fixed order mixed controller. In another study, we looked into the problem of designing equalizers for communication channels from an H_∞ point of view.

Summary of the most important research findings

1 Partially Structured Matrices and Numerically Reliable Displacement Algorithms

Many problems in multichannel digital filtering, one and two-sided linear prediction, signal and image processing, $\mathcal{H}_\infty$-control and in many other areas of physical sciences and engineering, can be formulated in terms of composite matrices involving sums of products of Toeplitz, Hankel, Toeplitz-plus-Hankel, Vandermonde, Cauchy, Schur-Cohn, Routh-Hurwitz matrices and their inverses. The current scientific literature encounters a large number of methods exploiting these structures to speed-up the computation and to suggest fast algorithms for solving related problems. Unfortunately, many of these algorithms have a theoretical importance only, since when implemented on digital computers, they often suffer from the propagation of roundoff errors, thus efficiently computing completely incorrect answers! For this reason all available mathematical software tools explore general (structure-ignoring) methods, and do not contain tool-boxes for structured matrix computations. But general methods have their own limitations, and often they are also not appropriate for practical purposes, because ignoring the structure artificially squares the amount of data, thus requiring unnecessary large storage and extremely large amount of the CPU time. In the last few years, we have made considerable progress in designing a wide variety of not only *fast*, but also *numerically accurate*, algorithms. A summary of this research is provided below.

1.1 Reliable Numerical Algorithms

In this section, we summarize a few of the important results we obtained regarding partially structured matrices and numerically reliable displacement algorithms.

- We designed new fast and accurate algorithms for solving Toeplitz-plus-Hankel-like linear equations. These algorithms seem to be the most practical (accuracy, speed, storage, amen-ability to parallel implementations) among all other available direct algorithms (most of such algorithms are limited to just Toeplitz matrices). It is shown that to fully exploit symmetry of the coefficient matrices one has to consider *partially reconstructible* matrices [defined via displacement operators with non-trivial kernels], otherwise the resulting algorithms are not accurate.
- When solving Toeplitz linear equations by the conjugate gradient method, there inevitably appears the problem of building up a good preconditioner. First preconditioners were circulant

matrices suggested by G.Strang and T.Chan (they were FFT based, i.e. *complex-arithmetic*). The idea to exploit special beneficial properties of *partially reconstructible* matrices is shown to be very fruitful to design new alternative *real-arithmetic* preconditioners. Indeed, we designed a family of eight new preconditioners of G.Strang type. These new preconditioners are based on any of the eight version of discrete cosine or sine transforms (i.e., real DCT or DST instead of complex FFT). The crucial clustering-of-the-spectrum property is established for all new preconditioners, guaranteeing a *superlinear convergence* for the preconditioned conjugate gradient method, [1,6].

- Our study of *partially reconstructible* matrices led us to formulate new accurate algorithm for the important class of *boundary* tangential interpolation problems, such as boundary tangential Nevanlinna-Pick problem.

- We compared the performance of various algorithms involving Vandermonde Matrices, and found that although the Traub algorithm had a reputation of a very inaccurate method, there is a variant of this algorithm which provides a remarkably high accuracy. This variant was generalized to the wider class of Vandermonde-like matrices, appearing in the inverse spectral problems of matrix polynomials, [4].

- We obtained a family of fast and very accurate algorithms for Cauchy matrices (of the Schur type as well as of the Björck-Pereyra type), and identified a class of *totally positive* matrices, for which we are able to achieve a remarkable high accuracy (even though the condition number can be reciprocal to machine precision, we are still able to compute the result with full possible relative accuracy), [7].

- These new algorithms for Vandermonde and Cauchy matrices admit an interpretation, as a classical and several new (polynomial and rational, updating and downdating) divided differences schemes. It is shown how pivoting can be efficiently implemented to achieve a smaller size for the backward errors for these algorithms, [8].

2 Generalized Inverse Scattering with Applications to Estimation, Control and Moment Problems

The research undertaken under this subcontract has helped us pursue extensions and applications of structured matrix computations to problems in $\mathcal{H}_\infty$ filtering, inverse scattering, adaptive filtering, and recursive updates. The research has also enabled us to explore new areas and, in particular, to develop robust estimation schemes for data fusion scenarios and to study the performance limits of several adaptive schemes.

2.1 Inverse Scattering and $\mathcal{H}_\infty$ Problems

As is well-known, the solution of $\mathcal{H}_\infty$ problems requires the determination of contractive operators that map certain input signals to certain output signals. Such operators, and tests for their contractiveness, arise naturally in a scattering formulation of the generalized Schur algorithm, which is an efficient procedure for the triangular factorization of matrices with displacement structure. In the articles [9,10], we have clarified this connection and have also shown how to reformulate $\mathcal{H}_\infty$ problems, both for the finite and the infinite horizon cases, in terms of equivalent factorization problems for positive-definite matrices with structure.

More specifically, the generalized Schur algorithm is a fast procedure for the factorization of matrices with displacement structure. A major feature of this algorithm is that it admits a powerful physical interpretation as a natural inverse scattering algorithm for determining the parameters of a cascade of elementary sections that combine together to form a layered medium structure; moreover, such scattering cascades map contractive loads at their outputs to contractive functions at their inputs. By studying the flow of signals through these layered media, several important (both old and new) results can be insightfully obtained. Among some of these we may mention the use of energy conservation ideas to obtain various matrix factorization and inversion formulas, layer-peeling and layer-adjoining algorithms for inverse scattering problems, and local blocking (or transmission zero) properties that combine together to yield cascades that satisfy global interpolation conditions.

In the articles [9,10], we demonstrated the usefulness of the contractive mapping property for $\mathcal{H}_\infty$ problems. It was shown that by properly defining a convenient matrix structure, both conditions for the existence of, and a recursive construction for, $\mathcal{H}_\infty$ solutions are quite directly obtained. The key was that, as just noted, the generalized Schur algorithm constructs a contractive mapping that relates two so-called wave operators (consisting of the input and output signals of the scattering cascade). This fact is exploited to solve $\mathcal{H}_\infty$ problems since these problems are essentially concerned with studying such contractive mappings.

One benefit of the formulation in [9] is that it provides a transparent connection between $\mathcal{H}_\infty$ problems and scattering cascades. By using the generalizations of [10], which showed how to introduce some nonlinear feedback reflections at the input of such cascades, it becomes possible to explore efficient solutions to some $\mathcal{H}_\infty$ problems with embedded nonlinearities. This issue is currently under investigation. In [11] we have also shown how to construct inverse scattering cascades with multi-input/multi-output ports that satisfy a maximum entropy property. Such cascades have received considerable attention in modern signal processing, especially since the work of Burg on spectral analysis.

2.2 Structured Matrix Embeddings and Subband Filters

We have further shown in [12] how insights from the study of structured matrix factorizations can be used to develop new structures for subband adaptive filtering.

As is known, frequency-domain and subband implementations improve the computational efficiency

and the convergence rate of adaptive schemes. The so-called multidelay adaptive filter (MDF) belongs to this class of block adaptive structures and is a DFT-based algorithm. In the article [12], we developed adaptive structures that are based on the trigonometric transforms DCT and DST, and on the discrete Hartley transform (DHT). As a result, these structures involve only real arithmetic, and are attractive alternatives in cases where the traditional DFT-based scheme exhibits poor performance. The filters are derived by first presenting a derivation for the classical DFT-based filter that allows us to pursue these extensions very immediately. The approach used in [12] also provides further insights into subband adaptive filtering.

More specifically, computational complexity is often a burden in applications that require long tapped-delay adaptive filter structures, such as acoustic echo cancellation where filters with hundreds or even thousands of taps are necessary to model the echo path. Frequency-domain and subband adaptive filters have been proposed to reduce the computational requirements inherent to such applications. These techniques not only result in more efficient structures (due to the use of efficient block signal processing methods) but they also improve the convergence rate of an adaptive algorithm (due to a decrease in the eigenvalue spread of the correlation matrix of the transformed signals). A well known example is the multidelay adaptive filter (MDF), which relies on the use of the discrete-Fourier transform (DFT).

The MDF structure has been derived in the literature in the DFT domain only. However, one would expect that different frequency domain transformations (other than the DFT) can result in different levels of performance (both computationally and otherwise), since performance is highly dependent on both the statistical properties of the input signals and on the nature of the frequency transformations. This fact motivated us to develop in [12] frequency-domain adaptive structures that are based on the trigonometric transforms DCT and DST, and on the discrete Hartley transform (DHT). It turns out that the traditional derivations of frequency-domain adaptive filters cannot be directly extended to these new signal transformations without some effort. For this reason, we first presented in [11] a new derivation for the classical DFT-based MDF structure by using a so called *embedding* approach. This approach allows us to exploit in a direct manner different forms of matrix-function structure.

The new adaptive filters turn out to be useful for applications where real arithmetic is required. Moreover, since efficient algorithms exist for computing the DCT, DST, and DHT, these schemes also lead to efficient adaptive filter structures.

2.3 Fast Updating of Structured Linear Systems

In related work, we have also addressed the problem of developing fast recursive procedures for updating the solutions of structured linear systems of equations [13]. These are equations whose coefficient matrices are structured and remain so after rank-one time-updates, and whose right-hand side terms also undergo changes in a certain structured manner. Such systems arise in several applications in signal processing and communications, as well as in adaptive filtering.

2.4 Robust Data Fusion

The support by this award has also enabled us to explore problems in related areas in filtering and estimation. In the article [14], we have formulated and solved a parameter estimation problem that shows how to combine, in a certain optimal and robust manner, measurements that arise from a finite collection of uncertain models. This scenario occurs, for example, in data fusion applications and in cases that involve systems that can operate under different failure conditions. An example in the context of macroscopic diversity in wireless cellular systems was also considered.

More specifically, it is well understood that modeling errors in data are common in practice and they can be due to several factors including the approximation of complex models by simpler ones, the introduction of experimental errors while collecting data, or even the presence of unmodeled or unknown effects. Regardless of their source, modeling errors can adversely affect the performance of otherwise optimal estimators.

In the work [14], a general cost function that allows for different levels and sources of bounded parametric uncertainties in the data has been studied. The cost function is based on a constrained game-type formulation and it has been shown to lead to a regularized least-squares solution; albeit one where the regularization parameter is constructed optimally from the nominal data and from the available information about the uncertainties. Applications in the context of robust data fusion and diversity in wireless systems were considered.

2.5 Filter Performance via Energy Arguments

Insights from the study of scattering cascades in digital filtering and structured matrix factorizations turn out to be useful in the study of adaptive filter performance as well. In earlier work, we have shown how by representing an adaptive filter as a feedback interconnection of two blocks: a lossless block in the forward path and a static or dynamic block in the feedback, and by studying the energy flow through this cascade, statements about filter stability, robustness, and steady-state performance can be obtained.

In the more recent work [15,16], with related articles that are currently under preparation, we have extended this unifying approach to the transient analysis of adaptive filters. In addition to deriving earlier results in a unified and comprehensive manner, the approach leads to new stability and performance results especially for adaptive schemes for which existing analyses are not complete. As mentioned above, the framework is based on energy-conservation arguments and it helps bypass several of the difficulties encountered by traditional approaches.

Adaptive filters are, by design, time-variant and nonlinear systems that adapt to variations in signal statistics and that learn from their interactions with the environment. The success of their learning mechanism can be measured in terms of how fast they can adapt to changes in the signal characteristics, and how well they can learn given sufficient time. It is therefore typical to measure the performance of an adaptive filter in terms of both its transient performance and its steady-state performance. The former is concerned with the stability and convergence rate of an adaptive scheme,

while the latter is concerned with the mean-square-error that is left in steady-state.

There have been extensive works in the literature on the performance of adaptive filters with many ingenious results and approaches. However, it is generally observed that most of these works study individual algorithms separately. This is because different adaptive schemes have different nonlinear update equations, and the particularities of each case tend to require different arguments and assumptions.

The contribution of [15,16], and related works, has been to study optimal selection for filter nonlinearity and to provide a unifying treatment of the transient analysis of adaptive filters in the linear-error case (the nonlinear error case is treated in a separate submission, which is currently under preparation). In so doing, it not only becomes possible to treat various algorithms uniformly and to derive earlier results in a comprehensive manner within this framework, but the framework itself also allows us to arrive at new stability results and at new performance expressions. In addition, it becomes also possible to handle algorithms for which complete analyses are not yet available in the literature. The approach adopted in [15,16] is based on studying the energy flow through each iteration of an adaptive filter, and it relies on a fundamental error variance (or energy conservation) relation. This approach was originally developed by Sayed & Rupp (1994) and used in the study of the robustness performance of adaptive filters within a deterministic framework. It was further used more recently by Yousef & Sayed (1999,2000) to provide a unifying approach to the steady-state performance of adaptive filters within a stochastic framework. In [16], it has been shown how to use the energy conservation relation for transient analysis, as opposed to steady-state analysis. Some subtleties arise here that require special handling than in steady-state analyses.

3 Design of Optimal mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Controllers

The mixed $\mathcal{H}_2/\mathcal{H}_\infty$ approach to controller design is an attempt to incorporate performance and guarantee robustness, arguably the most desirable properties, into a single controller. In $\mathcal{H}_2$ design a quadratic performance criterion is minimized under the assumption that an underlying model of the system and the statistical nature of the optimal $\mathcal{H}_2$ controllers is susceptible to model uncertainties. In the $\mathcal{H}_\infty$ approach such assumptions are not made, and a mimimax type-of criterion is adopted; however, the solutions may be over-conservative in terms of $\mathcal{H}_2$ performance. Fortunately, given an upper bound on the $\mathcal{H}_\infty$ performance there is a multiplicity of $\mathcal{H}_\infty$ controllers, and this allows the possibility of seeking a mixed controller that achieves the optimal $\mathcal{H}_2$ performance subject to an $\mathcal{H}_\infty$ bound.

Despite recent advances in multivariable robust control theory, the robust performance problem formulated in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ framework largely remains an open problem. One problem is that while the $\mathcal{H}_2$ and the $\mathcal{H}_\infty$ problems have readily computable Riccati-equation-based analytic solutions, there is no known analytic solution for the mixed problem. Furthermore, conceivable numerical algorithms are computationally intensive. In this study, using a number of ideas from convex optimization

theory, we have developed an efficient numerical approach to design fixed-order mixed controllers.

One difficulty with the optimal mixed problem is that it might not have a bounded order solution, and hence, it is hard to formulate the mixed problem as a finite dimensional optimization problem. We overcome this difficulty by restricting the search to controllers of fixed-order. This results in a constrained minimization problem that is non-convex in the system matrices of the fixed order controller. We develop iterative algorithms to numerically solve this minimization problem. The key idea in the derivation of these algorithms is to break up the non-convex problem into a series of easily solvable subproblems; each of which yields a suboptimal mixed controller with improved $\mathcal{H}_2$ performance, thus guaranteeing a monotonic decrease in the $\mathcal{H}_2$ norm in each iteration. Through numerical examples, we show that these algorithms yield controllers with $\mathcal{H}_2$ performance superior to that of the central controllers.

4 Equalization of Communication Channels from an H_∞ point of view

We approach the equalization of communication channels from an H_∞ estimation theoretic point of view. The richness of the robust H_∞ estimation theory and of its stochastic interpretation risk sensitive estimation has been the basic motivation for this approach. The results obtained in this attempt provide us with a new and different perspective for the understanding and analysis of the equalization problem, as well as for H_∞ estimation itself.

We first concentrate on the linear equalization problem and present the formulation and the characterization of the H^∞ linear equalizers for the most general multiuser and multibranch receiver setup. The basic results that we obtained under this branch underline the importance of the minimum phase property of the communication channels. H_∞ theory also prescribes the methods for dealing with non-minimum phase channels. Among these methods, we investigate the merit of delay in the equalization process. An important result in this area is that the minimum delay required in the equalization of non-minimum phase channels is equal to the number of non-minimum phase zeros of the channel. We also compare the linear H^∞ equalizer with its minimum mean square errror(MMSE) counterpart and illustrate the performance improvement in terms of the worst case BER measure.

As the second major subject, we look at the nonlinear decision feedback equalization (DFE) problem again from an H_∞ perspective. One major result is the H_∞ optimality of the MMSE-DFE under the standard correct decisions assumption, which forms a non-trivial example where the H_2 and H_∞ theories coincide. The H_∞ based approach is also able to provide a solution for the incorrect decisions assumption; this is made possible by the deterministic nature of the H_∞ theory and again illustrates another possible benefit of our approach.

Finally, we look at finite impulse response (FIR) equalization. We show that the design of the FIR risk sensitive and the mixed H_2/H_∞ equalizers can be posed as convex optimization problems which can be efficiently solved using the interior point algorithms.

List of Scientific Publications

1. T.Kailath and V.Olshevsky, *Displacement structure approach to discrete-trigonometric transform based preconditioners of G.Strang and T.Chan types,* Calcolo, 33(1996), 191-208.

2. T.Kailath and V.Olshevsky, *Binch-Kaufman pivoting for partially reconstructible Cauchy-like matrices with applications to Toeplitz-like linear systems, and to boundary rational matrix interpolation problems*, Linear Algebra Appl.,261(19970,251-302.

3. T.Kailath and V.Olshevsky, *Displacement structure approach to polynomial Vandermonde and related matrices*, Linear Algebra Appl., 261(1997), 49-90.

4. I.Gohberg and V.Olshevsky, *On the generalized Parker-Traub algorithm for inversion of Vandermonde and related matrices*, to appear in Journal of Complexity, 1997.

5. T.Kailath and V.Olshevsky, *Unitary Hessenberg matrices and fast generalized Parker-Forney-Traub and Björck-Pereyra algorithms for Szegö-Vandermonde matrices*, submitted to Linear Algebra Appl.

6. T.Kailath and V.Olshevsky, *Displacement structure approach to discrete-trigonometric-transform based preconditioners of G.Strang and T.Chan types,* submitted to SIAM J. Matrix Analysis and Appl.

7. T.Boros, T.Kailath and V.Olshevsky, *A fast Björck-Pereyra-type algorithm for Cauchy linear equations,* submitted to NUmerical Mathematik.

8. T.Boros, T.Kailath and V.Olshevsky, *Predictive pivoting and backward stability of fast Caucy solvers*, submitted to SIAM J. on Matrix Analysis and Appl.

9. T. Constantinescu, A. H. Sayed, and T. Kailath, "Displacement structure and $\mathcal{H}^{\infty}$ problems," in *System Theory: Modeling, Analysis and Control*, T. Djaferis and I. Schick, editors, pp. 419–432, Kluwer, MA, 2000.

10. T. Constantinescu, A. H. Sayed, and T. Kailath, "Generalized inverse scattering experiments and structured matrix inequalities," to be submitted to special issue of *Linear Algebra and Its Applications* on *Structured and Infinite Systems of Linear Equations.*

11. A. H. Sayed, T. Constantinescu, and T. Kailath, "Recursive construction of multi-channel transmission lines with a maximum entropy property," to appear as a book chapter.

12. R. Merched and A. H. Sayed, "An embedding approach to frequency-domain and subband adaptive filtering," to appear in *IEEE Transactions on Signal Processing,* 2000.

13. S. Chandrasekaran, M. Gu, and A. H. Sayed, "Fast updating of structured linear systems of equations with applications in adaptive filtering," *Proc. Asilomar Conference on Signals, Systems, and Computers*, pp. 427–431, CA, Oct. 1999.

14. A. H. Sayed, T. Y. Al-Naffouri, and T. Kailath, "Robust estimation for uncertain models in a data fusion scenario," to appear in *Proc. IFAC System Identification Symposium*, Santa Barbara, CA, June 2000.

15. T. Y. Al-Naffouri, A. H. Sayed, and T. Kailath, "On the selection of optimal nonlinearities for stochastic gradient adaptive algorithms," to appear in *Proc. ICASSP*, Istanbul, Turkey, June 2000.

16. T. Y. Al-Naffouri, A. H. Sayed, and T. Kailath, "Transient analysis of adaptive filters via energy conservation arguments – Part I: The data nonlinearity case," *to be submitted.* [Other related articles are currently under preparation for near-future submission.]

17. Alper T. Ergodan, Babak Hassibi, and Thomas Kailath, "FIR H_∞ Equalization," *Proc. ICASSP*, Istanbul, Turkey, June 2000.

18. Alper T. Ergodan, Babak Hassibi, and Thomas Kailath, "Decision Feedback Equalization from an H_∞ perspective," *to be submitted.*

19. T. Kailath, A. H. Sayed, and B. Hassibi, *Linear Estimation*, Prentice-Hall, NJ, 854pp, 2000 (a graduate-level textbook).

20. T. Kailath and A. H. Sayed, editors, *Fast Reliable Algorithms for Matrices with Structure*, SIAM, PA, 342pp, 1999 (an edited volume).

21. B. Hassibi, A. H. Sayed, and T. Kailath, *Indefinite Quadratic Estimation and Control: A Unified Approach to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Theories*, SIAM, PA, 555pp, 1999 (a research monograph).

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